Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

Understanding Electrical Properties of Dielectrics

By

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Abstract: This presentation is a tutorial on electrical properties of dielectrics for the benefit of students. It starts with a review of necessary electromagnetic preliminaries including flux and Gauss' law in free-space. Polarization in dielectrics is then discussed, leading to the generalized Gauss' law. Reviewing what makes permittivity complex in lossy materials, the presentation also considers why dielectrics delay electromagnetic waves. A brief introduction to tensors, and a quick look at anisotropic dielectrics follow.

Keywords: Polarization in dielectrics, Permittivity, Complex permittivity, Delay in dielectrics, Tensors, Artificial dielectrics

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Krishnasamy T. Selvan obtained his BE (Hons), MS and PhD degrees respectively from Madurai Kamaraj University (1987), Birla Institute of Technology and Science (1996) and Jadavpur University (2002). He also obtained a PGCHE in Higher Education from University of Nottingham in 2007.

Selvan has been a Professor in the Department of Electronics and Communication Engineering, SSN College of Engineering, India, since June 2012. From early 2005 to mid-2012, he was with the Department of Electrical and Electronic Engineering, University of Nottingham Malaysia Campus. He also held the positions of the Assistant Director of Teaching and Learning for the Faculty of Engineering and the Deputy Director of Studies of the Department of Electrical and Electronic Engineering.

From early 1988 to early 2005, Selvan was with SAMEER – Centre for Electromagnetics, Chennai, India. During 1994–1997, he was the Principal Investigator of a collaborative research programme that SAMEER had with the National Institute of Standards and Technology, USA. Later he was the Project Manager/Leader of some successfully completed antenna development projects.

Selvan's professional interests include electromagnetics, antenna metrology, horn antennas, printed antennas, and electromagnetic education. In these areas, he has authored or coauthored a number of journal and conference papers. Selvan was on the editorial boards of the International Journal of RF and Microwave Computer-Aided Engineering and the International Journal on Antennas and Propagation. He has been a reviewer for major journals including the IEEE Transactions on Antennas and Propagation. He was technical programme committee co-chair for the IEEE Applied Electromagnetics Conference held in Kolkata in December 2011, and Student Paper Contest co-chair for IEEE AEMC 2013 to be held in Bhubaneswar. He was Publications Chair for the IEEE MTT-S International Microwave and RF Symposium (IMaRC) held in Bangalore in December 2014. He co-organized sessions on EM/microwave education during IMaRC 2014 and International Symposium on Antennas and Propagation, Kochi, 2014.

Selvan founded the Madras Chapter of the IEEE Antennas and Propagation Society (AP-S) in 2013, and served as its Chair till 2015. He is a member of the Education Committee of the IEEE Antennas and Propagation Society. He is an IEEE AP-S Region 10 Distinguished Speaker for 2015-16.

Selvan is a senior member of the IEEE, a Fellow of the Higher Education Academy (UK), and a Life Member of the Society of EMC Engineers (India).

Understanding Electrical Properties of Dielectrics

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Objectives

- **Specific:**
  - Quick review of some EM preliminaries
  - Electric field in materials – physical and mathematical ideas
  - Dielectric polarization and permittivity
  - Artificial dielectrics

- **Broad:**
  - Encouraging continued development of conceptual understanding by self-study

Expected learning

- Physical and mathematical ideas of polarization in dielectrics
- The meaning of dielectric constant and loss tangent
- Delay in dielectrics
- The idea of artificial dielectrics

Importance of the topic

- Good conceptual understanding *always* helps with teaching, research and scholarship
- Material electromagnetics becoming increasingly important
- Printed antennas

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SOME PRELIMINARIES

EM constants & material properties

- c, velocity of light, with the modern value of $c = 2.997930 \times 10^8$ m/s

- In SI units:
  - ε_o, electric constant
  - μ_o, magnetic constant

- Material properties:
  - ε_r
  - μ_r
  - σ

Fundamental EM Quantities

- Sources:
  - Charge density (ρ) – volume, surface, line
  - Conduction current density (**J**)
- Desired quantities:
  - Electric field intensity (**E**)
  - Electric polarization (**P**)
    - **Electric flux density D** = $\boldsymbol{\varepsilon_o \varepsilon_r}$ **E** = $\boldsymbol{\varepsilon}$**E**

Relative permittivity, representing polarization

  - Magnetic flux density (**B**)
  - Magnetization (**M**)
    - Magnetic field intensity **H** = **B**/μ

Volume charge density

$\rho_v \rightarrow$ volume charge density

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} = \frac{dq}{dv}$$

$$dq = \rho_v dv$$

$$q = \int_v \rho_v dv$$

http://apollo.lsc.vsc.edu/classes/met130/notes/chapter14/charge_distribution.html

Surface and line charge densities

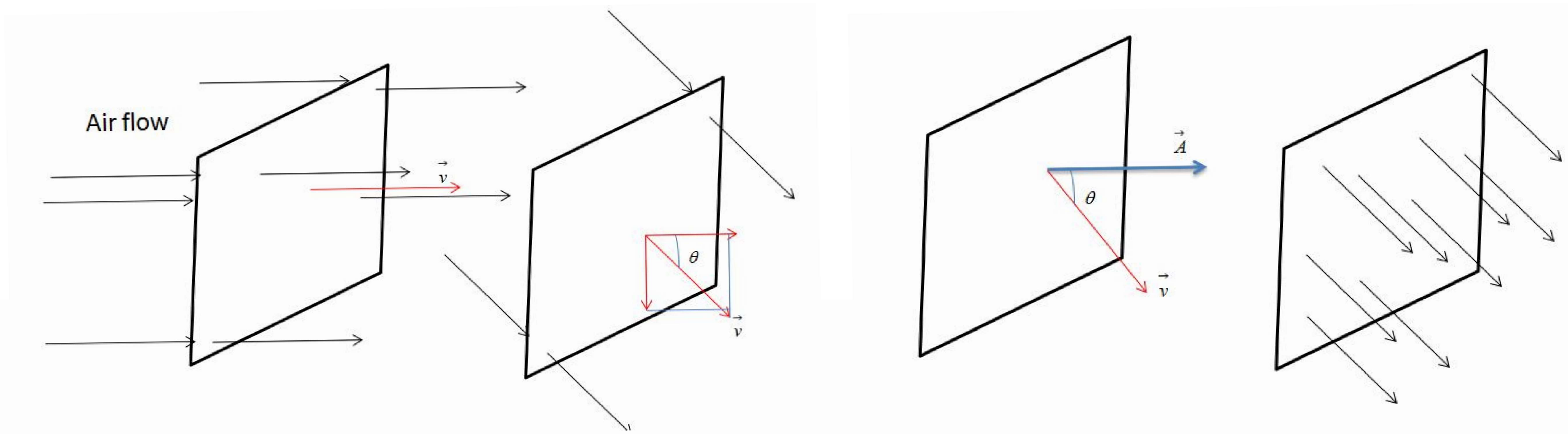
- Surface charge density:

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s} = \frac{dQ}{ds} \quad \mathrm{C/m^2}$$

- Line charge density:

$$\rho_l = \lim_{\Delta l \to 0} \frac{\Delta Q}{\Delta l} = \frac{dQ}{dl} \quad \mathrm{C/m}$$

What is Flux?



Redrawn based on [5], Fig. 23-2, p. 606

- 'Flow' of field through a surface
  - Flux in Latin means 'to flow'
- Flux of the velocity field through the loop:

$$\Phi = (v\cos\theta)S = \mathbf{v}\cdot\mathbf{S}$$

Gauss' Law in free space

- Statement:
  - The outward flux of the electric field intensity vector through any closed surface in free space is equal to the total charge enclosed by that surface, divided by ε_o.

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q_{enc}}{\varepsilon_o} \qquad (1)$$

  - In terms of volume charge density:

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_o} \int_V \rho dv \qquad (2)$$

Maxwell's equations

Integral and differential forms

$$\oint_s \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_o}\int_v \rho dv \qquad \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_o}$$

$$\oint \mathbf{E}\cdot d\mathbf{L} = -\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S} \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_o\varepsilon_o\frac{d\Phi}{dt} + \mu_o i_{inc} \qquad \nabla\times\mathbf{B} = \mu_o\mathbf{J} + \mu_o\varepsilon_o\frac{\partial\mathbf{E}}{\partial t}$$

Maxwell's equations

Phasor form

- Phasor forms useful for linear systems with periodic time-function excitation
  - Can convert integro-differential equations to algebraic equations
  - $\frac{\partial}{\partial t}$ can be replaced by $j\omega$ in differential equations
- Maxwell's curl equations:

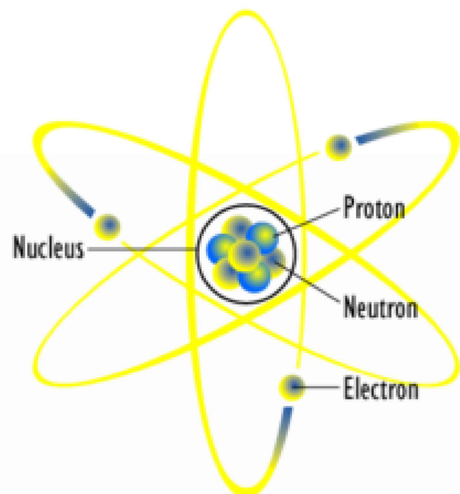
$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}$$

$$\nabla \times \mathbf{B} = \mu \mathbf{J} + j\omega\mu_o\varepsilon_o \mathbf{E}$$

DIELECTRICS – ELECTRICAL PROPERTIES

The atom

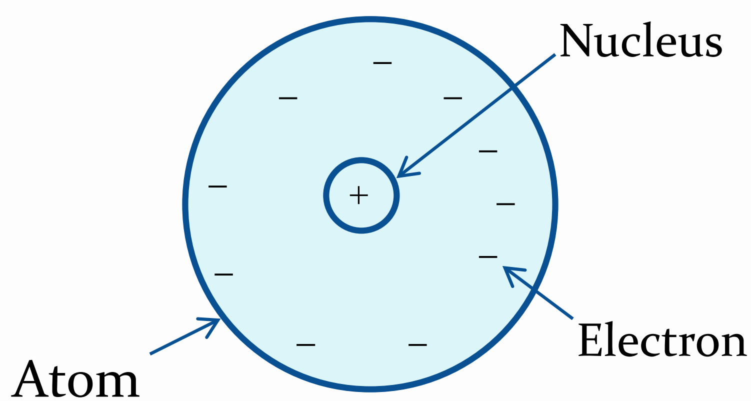
- An atom contains a very small but massive nucleus
  - Nucleus contains neutrons and protons



https://en.wikipedia.org/wiki/Rutherford_model

- Negatively charged electrons revolve about the nucleus
- All matter made up of one or more of the 118 different elements

- The molecule is the smallest constituent of elements and compounds
- Atoms and molecules macroscopically neutral in a dielectric

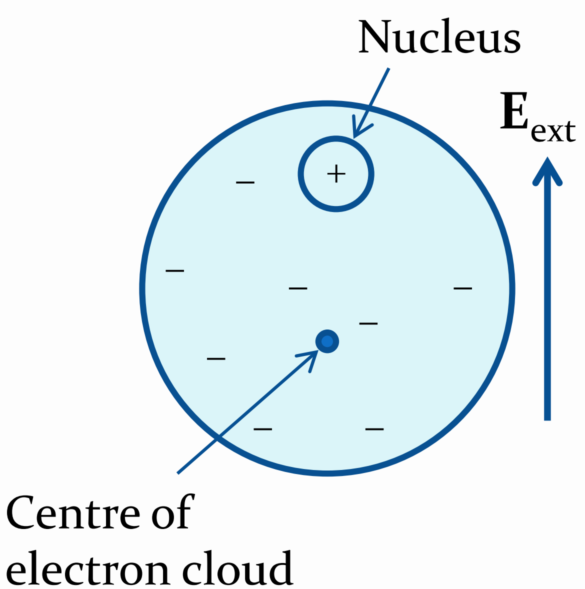


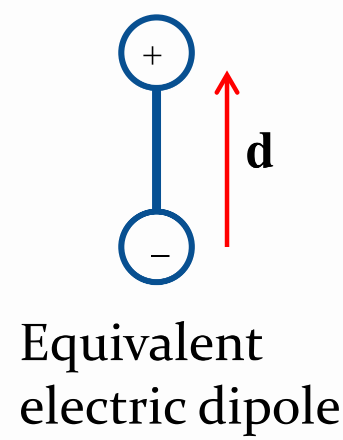
Dielectrics – quick facts

- Also called insulators
- Contain bound positive and negative charges held in place by atomic and molecular forces
- No free charges
- Every point reacts to an external field
  - by stretching or rotating, but otherwise unaffected
  - in direct proportion to the field in a **linear** material
  - in the same way in a **homogeneous** material
  - independent of the field's direction in an **isotropic** material

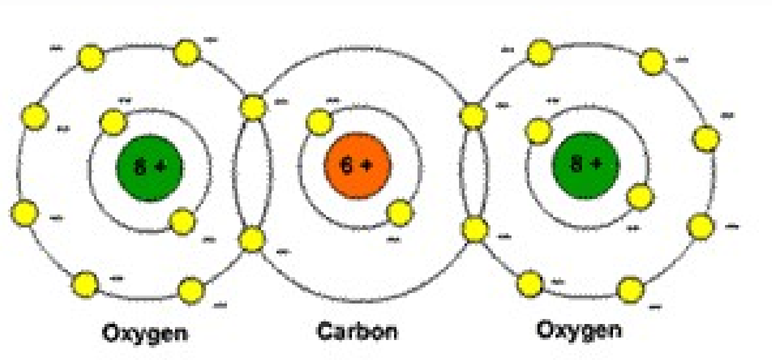
Polarization of dielectrics

- **NON-POLAR MATERIALS:**
  - When an external E field is applied, the bound charges q and $-q$ shift positions slightly
  - A dipole is thus formed:





- Examples of non-polar materials: O_2, N_2, H_2, CO_2



Non-polar CO_2

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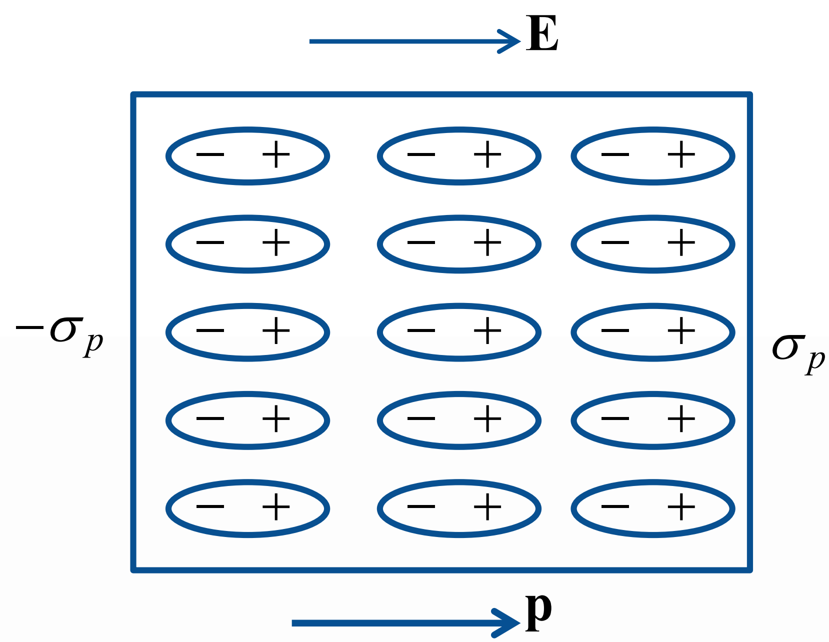
http://www.school-for-champions.com/chemistry/polar_molecules.htm#.VwyYlnrFvIW



A dielectric medium polarized by an electric field

- Moment of equivalent dipole:

$$\mathbf{p} = q\mathbf{d}$$

- Forces act on the charges:

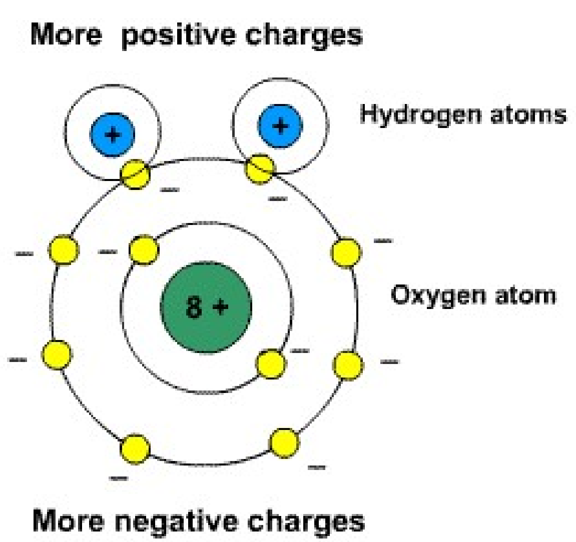
$$\mathbf{F}_{e1} = q\mathbf{E}_{ext} \qquad \mathbf{F}_{e2} = -q\mathbf{E}_{ext}$$

- **p** and $\mathbf{E}_{ext}$ are collinear and have same direction

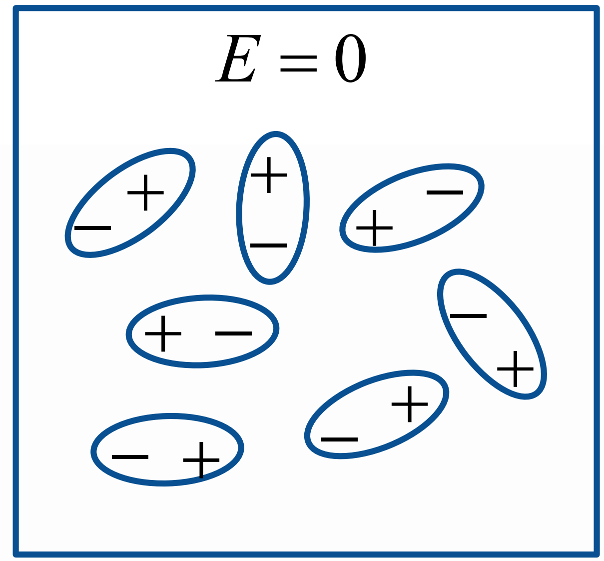
- **d** very small, smaller than the dimensions of atoms and molecules

- Thus charges cannot separate from one another and move

- **POLAR MATERIALS:**
  - Have randomly oriented, permanent dipole moments
  - Examples: HCl, H_2O, CO_2, NO_2



Permanent dipole moment in water molecule.



Polar material

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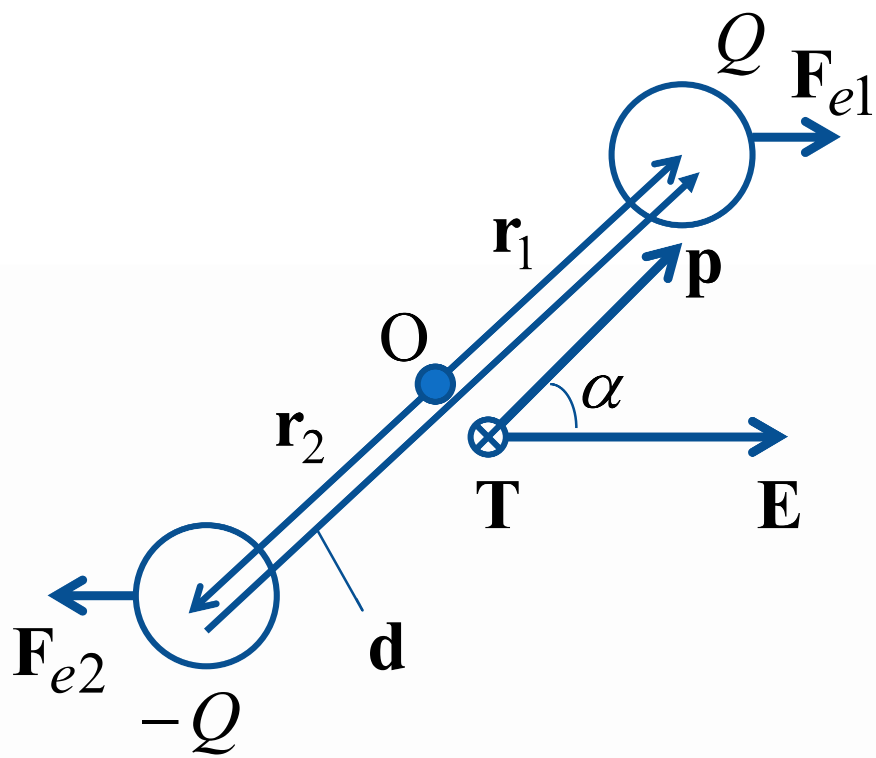
http://www.school-for-champions.com/chemistry/polar_molecules.htm#.VwyYlnrFvIW

- Torques act on the dipole are when $\mathbf{E}$ is present:

$$\mathbf{T}_1 = \mathbf{r}_1 \times \mathbf{F}_{e1} \qquad \mathbf{T}_2 = \mathbf{r}_2 \times \mathbf{F}_{e2}$$

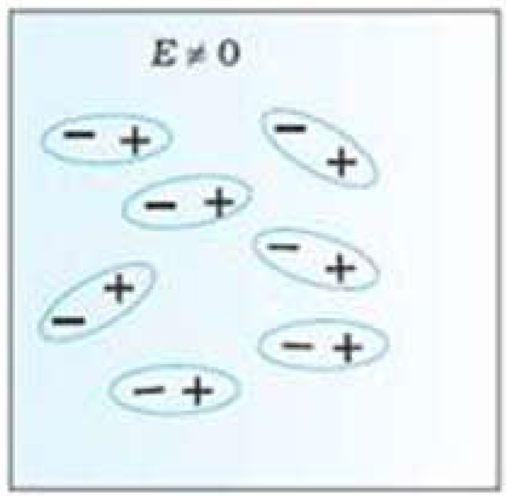


A dipole in a polar dielectric in electric field.
Redrawn based on [1], Fig. 2.1, p. 81

- Assuming uniform $\mathbf{E}$ along the dipole, net torque is

$$\mathbf{T}_d = \mathbf{T}_1 + \mathbf{T}_2 = q(\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{E}_{\text{ext}} = q\mathbf{d} \times \mathbf{E}_{\text{ext}} = \mathbf{p} \times \mathbf{E}_{\text{ext}}$$

- These torques tends to rotate the dipoles and attempts to align them in the direction of the field:



Polar material in an electric field.

- The stronger the filed, the larger is the resultant dipole moment $\Sigma \mathbf{p}$ of all molecules

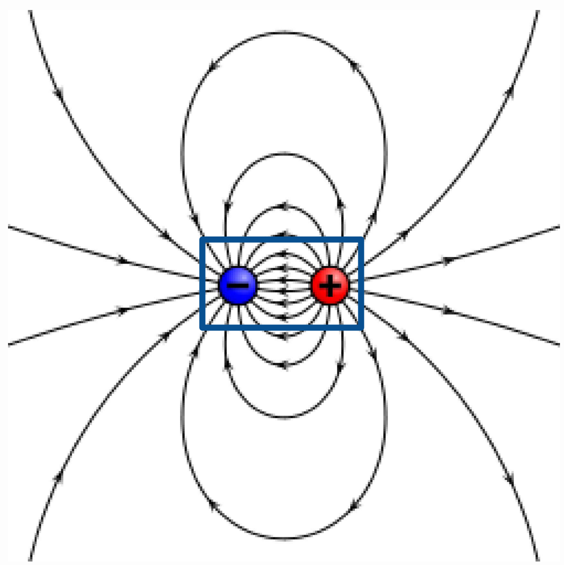
- Polarization of a dielectric is thus the process that, in the presence of an external electric field,
  - Makes atoms and molecules behave as dipoles aligned in the direction of the field in non-polar materials;
  - Orients the permanent dipoles in polar materials toward the direction of the field

- Removal of field makes the dielectric return to unpolarized state in most materials
  - Dielectrics called *electrets* remain polarized even after field removal – eg. Strained piezoelectric crystal

http://photonicswiki.org/index.php?title=Polarization_and_Polarizability

Polarization vector

- A polarized dielectric is a source of its own electric field
- The total field at any point in space is a sum of the primary field and the secondary field



- To facilitate computations, the dielectric can be modeled as a collection of equivalent dipoles in vacuum

- To characterize this model, a macroscopic quantity called polarization vector **P** is introduced as follows:

$$\mathbf{p}_{\mathrm{av}} = \frac{\left(\sum \mathbf{p}\right)_{\mathrm{in}\,\mathrm{d}v}}{N_{\mathrm{in}\,\mathrm{d}v}}$$

$$N_v = \frac{N_{\mathrm{in}\,\mathrm{d}v}}{dv}$$

Concentration of dipoles

- Polarization vector is defined as

$$\boxed{\mathbf{P} = N_{\mathrm{v}}\mathbf{p}_{\mathrm{av}} = \frac{\left(\sum \mathbf{p}\right)_{\mathrm{in}\,\mathrm{d}v}}{\mathrm{d}v}}$$

  - Unit for **P** is C/m^2

- It can be shown that

$$q_{\mathrm{p}s} = -\oint_S \mathbf{P} \cdot d\mathbf{S}$$

  - The total outward flux of **P** through a closed surface containing a dielectric is equal to the total bound charge enclosed by that surface, multiplied by –1

- In general, in any dielectric material **P** at a point is a function of **E** at that point:

$$\mathbf{P} = \mathbf{P}(\mathbf{E}) \tag{3}$$

- For linear materials in which *every point reacts to the field in direct proportion to the field,*

$$\mathbf{P} = \chi_e \varepsilon_o \mathbf{E} \tag{4}$$

Electric susceptibility. A pure, non-negative number, ≥ 0

$\chi_e = 0$ (vacuum)
$\chi_e \approx 0$ (air)

Generalized Gauss' law

- In a general electrostatic system containing both conductors and dielectrics, both free and bound charges are the sources:

$$\oint_s \mathbf{E} \cdot d\mathbf{S} = \frac{q_s + q_{\mathrm{ps}}}{\varepsilon_o}$$

$$\Rightarrow \qquad \oint_s \varepsilon_o \mathbf{E} \cdot d\mathbf{s} + \oint_s \mathbf{P} \cdot d\mathbf{s} = q_s$$

$$\Rightarrow \qquad \oint_s (\varepsilon_o \mathbf{E} + \mathbf{P}) \cdot d\mathbf{s} = q_s$$

- Defining a new quantity **D** called electric flux density,

$$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P} \tag{5}$$

we can rewrite the previous equation as

$$\boxed{\oint_S \mathbf{D} \cdot d\mathbf{s} = q_s} \tag{6}$$

  - Generalized Gauss' law
- Flux of **D** is termed the electric flux Ψ. Thus,

$$\psi = q_s$$

- In more general terms,

$$\boxed{\oint_s \mathbf{D} \cdot d\mathbf{s} = \int_v \rho dv} \qquad (7)$$

- Employing the divergence theorem, we get the differential form of Gauss' law:

$$\boxed{\nabla \cdot \mathbf{D} = \rho} \qquad (8)$$

Characterization of dielectric materials

- In general:

$$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}(\mathbf{E}) = \mathbf{D}(\mathbf{E}) \qquad (9)$$

- For linear, homogeneous dielectrics,

$$\boxed{\mathbf{D} = \varepsilon_o (\chi_e + 1)\mathbf{E} = \varepsilon_r \varepsilon_o \mathbf{E}} \qquad (10)$$

Relative permittivity of medium

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad (11)$$

Permittivity of the medium (F/m)

- If there are N molecules per unit volume, **P** can also be written as

$$\mathbf{P} = \varepsilon_o \chi_e \mathbf{E} = N\alpha_T \mathbf{E}_{loc} = N\alpha_T g\mathbf{E} \qquad (12)$$

  - α_T = molecular polarizability
  - g = ratio between local field and the applied field

- Total molecular polarizability arises due to the following atomic or molecular effects:
  - Electronic polarization
  - Ionic polarization
  - Permanent dipole contribution

- By using (10) and (12) we can get

$$\varepsilon_r = 1 + \frac{N\alpha_T g}{\varepsilon_o} = 1 + \chi_e \qquad (13)$$

- When the surrounding molecules act in a spherically symmetric fashion, the above equation can be shown to be

$$\boxed{\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_T}{3\alpha_o}} \qquad (14)$$

  - This is called Clausius-Mossotti relation
  - With frequency effects included, it becomes the Debye equation

- Dielectric constant, or relative permittivity, is dimensionless

$$\varepsilon_r = \chi_e + 1 \qquad (15)$$

  - $\varepsilon_r \geq 1$

- For free space and non-dielectric materials such as metals,

$$\boxed{\mathbf{D} = \varepsilon_o \mathbf{E}} \qquad (16)$$

Complex permittivity

- In a simple, non-conducting source-free medium, Maxwell-Ampere law in phasor form is:

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \tag{17}$$

- If the medium is conducting, there would be a current flowing:

$$\mathbf{J} = \sigma\mathbf{E} \tag{18}$$

- Maxwell-Ampere law in this case becomes

$$\begin{aligned}\nabla \times \mathbf{H} &= (\sigma + j\omega\varepsilon)\mathbf{E} \\ &= j\omega\left(\varepsilon + \frac{\sigma}{j\omega}\right)\mathbf{E} \qquad (19) \\ &= j\omega\varepsilon_c\mathbf{E}\end{aligned}$$

with the complex permittivity ε_c given by

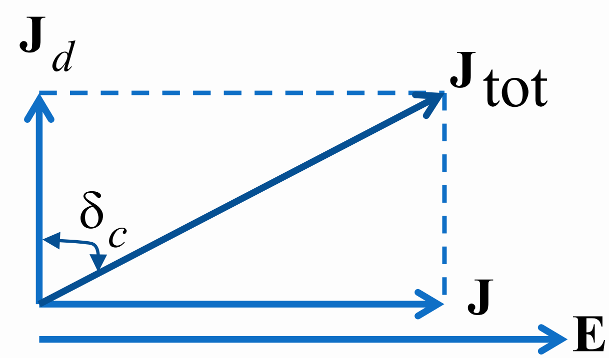
$$\begin{aligned}\varepsilon_c &= \varepsilon - j\frac{\sigma}{\omega} \quad \text{(F/m)} \\ &= \varepsilon' - j\varepsilon''\end{aligned}$$

- The imaginary part in the complex permittivity accounts for
  - damping due to out-of-phase polarization
  - ohmic losses

- Loss tangent, a measure of power loss in the medium, is given by

$$\boxed{\tan\delta_c = \frac{\varepsilon''}{\varepsilon'} \approx \frac{\sigma}{\omega\varepsilon}} \quad (20)$$

Ratio of amplitudes of conduction and displacement current densities



Delay in dielectrics with $\varepsilon_r > 1$

- We have deduced this equation earlier:

$$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P} = \varepsilon_o \varepsilon_r \mathbf{E} \qquad (21)$$

- From this equation,

$$\mathbf{P} = \varepsilon_o (\varepsilon_r - 1)\mathbf{E} \qquad (22)$$

- When an EM field impinges on the dielectric, the following currents are NOT induced:
  - conduction current, as there are no free charges,
  - magnetic current density, as the material is non-magnetic

- However, a polarization current is induced and is given by

$$\mathbf{j} = \frac{\partial \mathbf{P}}{\partial t} \tag{23}$$

- Maxwell-Ampere law now takes the form

$$\nabla \times \mathbf{B} = \mu_o \, \mathbf{j} + \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t} \tag{24}$$

- Use of (22) and (23) in (24) leads to

$$\nabla \times \mathbf{B} = \mu_o \, \varepsilon_o (\varepsilon_r - 1) \frac{\partial \mathbf{E}}{\partial t} + \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

$$= \mu_o \, \varepsilon_o \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} = \frac{\varepsilon_r}{c^2} \frac{\partial \mathbf{E}}{\partial t} \tag{25}$$

- Compare this equation with Maxwell-Ampere law for free space:

$$\nabla \times \mathbf{B} = \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \qquad (26)$$

- Thus (25) is the same as (26) except for this following replacement:

$$c \rightarrow \frac{c}{n}$$

  - $n = \sqrt{\varepsilon_r}$ is the refractive index of the medium

- EM waves thus propagate slower in a dielectric medium than in vacuum by a factor n (for $n > 1$)

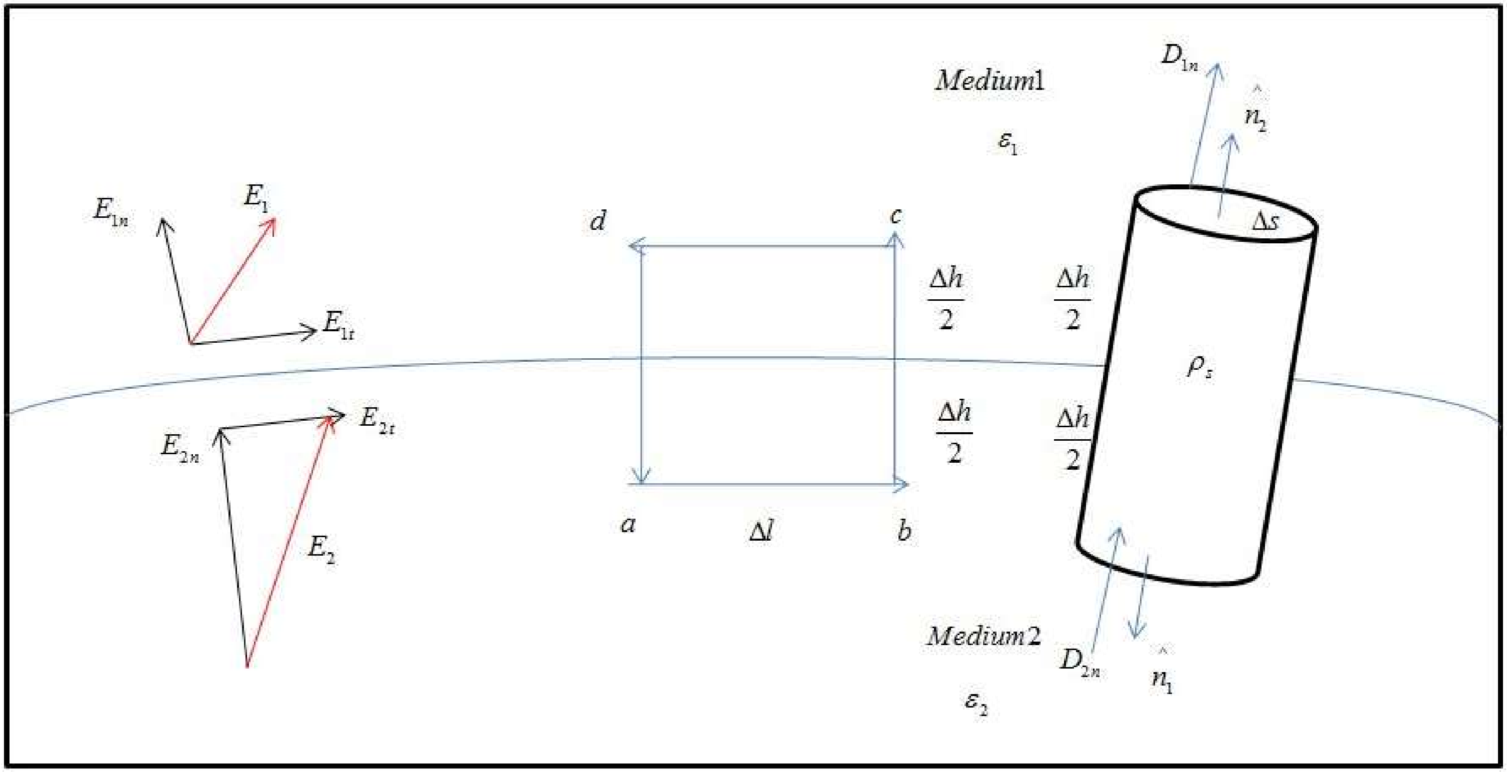
Dielectric breakdown

- ‘Normal’ dielectric behaviour obtains until the electric field intensity is within a threshold value
- If a certain value, called *dielectric strength*, is exceeded, the dielectric breaks down and ceases to be an insulator

Material	ε_r	Dielectric strength, MV/m
Air	1.0006	3
Polystyrene	2.6	20
Glass	4.5 – 10	25 – 40
Quartz	3.8 – 5	30
Bakelite	5	20
Mica	5.4 – 6	200

ε_r and dielectric strength of some materials [3]

Boundary conditions at dielectric-dielectric interface



Redrawn based on [3], Fig. 4-18, p. 178

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$
$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$$
$$\Delta h \to 0$$

- Tangential components of **E** and **D**:
  - Conservative property of electrostatic field employed

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = \int_a^b \mathbf{E}_2\cdot d\mathbf{l} + \int_c^d \mathbf{E}_1\cdot d\mathbf{l} = 0$$

$$E_{2t}\Delta l - E_{1t}\Delta l = 0$$

$$\boxed{E_{1t} = E_{2t} \quad (\mathrm{V/m})}$$

Tangential component of **E** is continuous across the boundary between any two media

$$\boxed{D_{1t} = \frac{\varepsilon_1}{\varepsilon_2} D_{2t}}$$

- Normal components of **E** and **D**:
  - Gauss law $\oint \mathbf{D} \cdot d\mathbf{S} = Q$ is employed, with $\Delta h \to 0$

$$\Delta Q = \rho_s \Delta S = D_{1n}\Delta S - D_{2n}\Delta S$$

$$\boxed{D_{1n} - D_{2n} = \rho_s}$$

The normal component of **D** changes abruptly at a charged boundary between any two media by an amount equal to the surface charge density

- If ρ_s is 0, then

$$\boxed{D_{1n} = D_{2n}}$$

$$\boxed{\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s}$$

Other than linear dielectrics

- Nonlinear dielectrics:
  - Constitutive relation between **D** and **E** nonlinear
  - χ_e and ε depend on E
- Ferroelectric materials:
  - Constitutive relation between **D** and **E** nonlinear
  - **D** also depends on the history of polarization
  - Eg., barium titanate, used in ceramic capacitors, ceramic filters, multiplexers

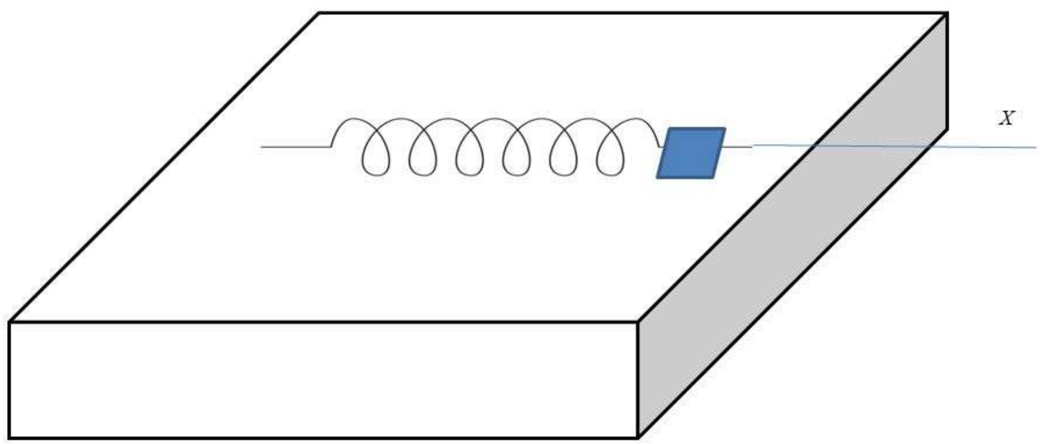
- Inhomogeneous materials:
  - ε is a function of spatial coordinates

- Anisotropic materials:
  - Individual components of **D** depend differently on different components of **E**
  - Thus, rather being a scalar as for linear materials, ε becomes a tensor

- Bi-isotropic and bi-anisotropic materials
  - Chiral materials

Brief introduction to tensors

- Consider a spring system as follows:

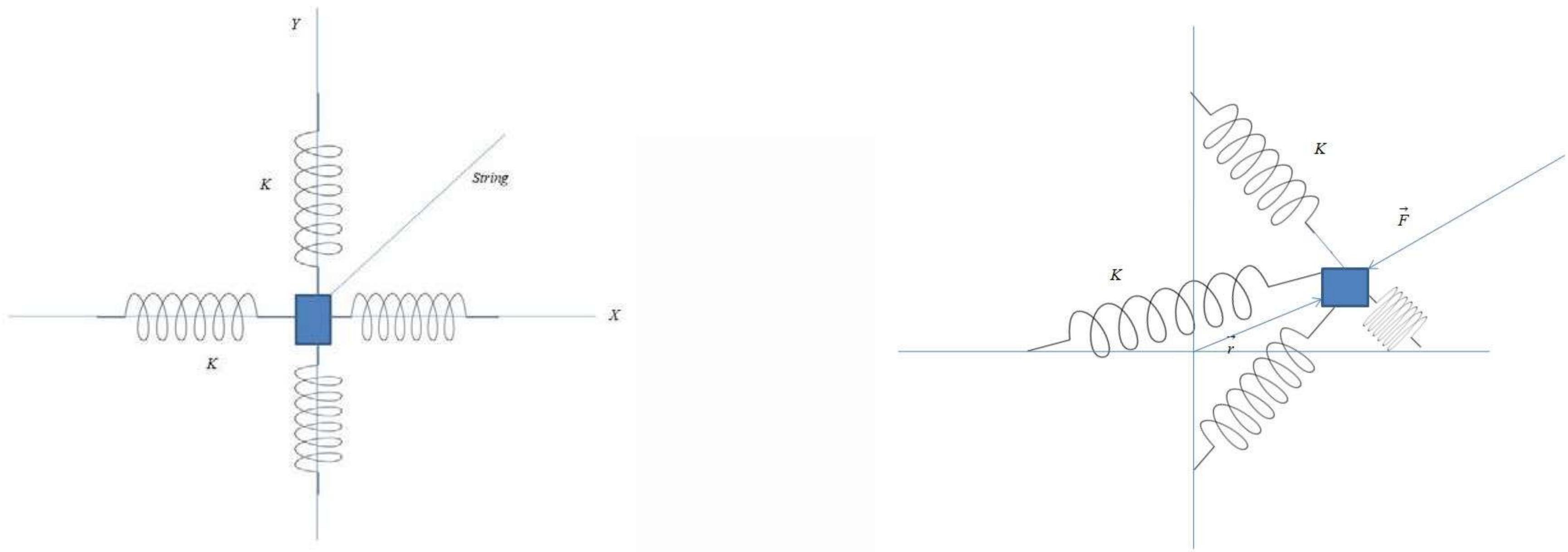


Redrawn based on [4], Fig. 5-22(a), p. 254

- Restoring force acting on the mass when the spring has elongated/compressed by an amount *x* is:

$$F = -\underbrace{\kappa}_{\text{Spring constant}} x$$

- Now consider a two-spring system attached at their centres and with a string tied at the common point:



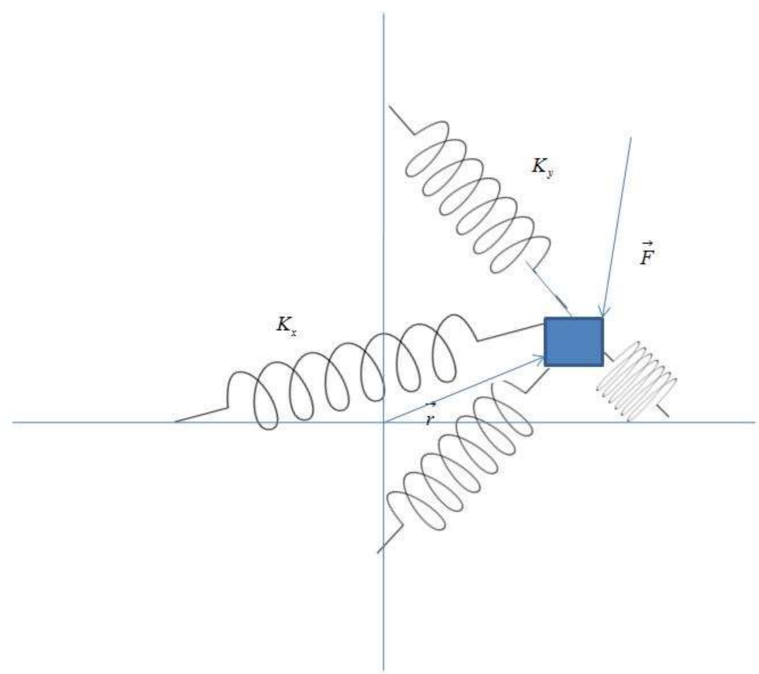
Redrawn based on [4], Fig. 5-22(b, c), p. 254

$$F_x = -\kappa x \qquad F_y = -\kappa y$$

- We can then write

$$\mathbf{F} = F_x\mathbf{a}_x + F_y\mathbf{a}_y$$
$$= -\kappa(x\mathbf{a}_x + y\mathbf{a}_y) = -\kappa\mathbf{r}$$

- Now consider that the two springs have different spring constants κ_x and κ_y:



Redrawn based on [4], Fig. 5-22(d), p. 254

- For this case, the force equation becomes

$$\mathbf{F} = -k_x x \mathbf{a}_x + k_y y \mathbf{a}_y$$

- In matrix form:

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = -\begin{bmatrix} \kappa_x & 0 \\ 0 & \kappa_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Represents vector

Tensor

Represents vector

$$\mathbf{F} = -\boldsymbol{\kappa}\mathbf{r}$$

- If the two springs are identical, then $\kappa_x = \kappa_y = \kappa$ and the above equation reduces to

$$\mathbf{F} = -\begin{bmatrix} \kappa & 0 \\ 0 & \kappa \end{bmatrix} \mathbf{r} = -\kappa \mathbf{r}$$

  - The tensor thus reduces to a scalar

- Can be readily generalized for three dimensions:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = -\begin{bmatrix} \kappa_x & 0 & 0 \\ 0 & \kappa_y & 0 \\ 0 & 0 & \kappa_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- All physical quantities are special forms of *second-rank* tensors, which are sets of 3^2 numbers
  - A first-rank tensor, or a vector, is a set of 3^1 numbers
  - A scalar is 3^0 numbers

- A more generalized form is obtained when the ends of the springs are attached to points off the coordinate axes
  - In such a case, each component of **F** depends on all the components **r**

- In this generalized case,

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = -\begin{bmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{xz} \\ \kappa_{yx} & \kappa_{yy} & \kappa_{yz} \\ \kappa_{zx} & \kappa_{zy} & \kappa_{zz} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Tensors encountered in EM usually look like this!

Anisotropic dielectrics

- Properties of isotropic media independent of direction
  - ε is scalar, and hence **D** and **E** are always collinear
- In anisotropic materials, electrical properties depend on the direction of the induced polarization
- Described in terms of matrix and tensor
- Thus:

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

- Crystalline dielectric materials generally anisotropic
- Periodic nature of crystals facilitates easier polarization along crystal axis rather than in other directions
- Examples:
  - Rutile (TiO_2) - ε_r 173 in direction parallel to crystal axis; 89 at right angles
  - Quartz: ε_r 4.7 – 5.1

Temperature dependence of ε_r

- Especially for polar materials, there is a strong dependence ε_r of on temperature
- This is caused by the effect of heat on the orientational polarization

Temperature, ° K	Approx. ε_r at 1 KHz	Approx. ε_r at 1 MHz
300	5.7	5.6
350	5.68	5.57
400	5.5	5.42
450	5.4	5.38
500	5.1	5.12
550	4.9	5
600	4.6	4.9

Glass fibre PCB substrate. *Journal of Korean Physical Society*, vol. 54, no. 3, March 2009, p. 1097.

Substrate choice for printed antennas

- Dielectric choice significantly impacts the performance
- Lower ε_r supports loosely bound fields and hence preferred for antennas
  - Higher values support tightly bound fields and thus suitable for other circuit components
- Higher ε_r results in reduced size for the given frequency
  - But higher ε_r also generally comes with higher loss tangent
- Tight dielectric constant tolerance desired for consistent centre-frequency performance

- Thicker substrate ensures more efficient radiation
  - But also increases feed radiation losses!

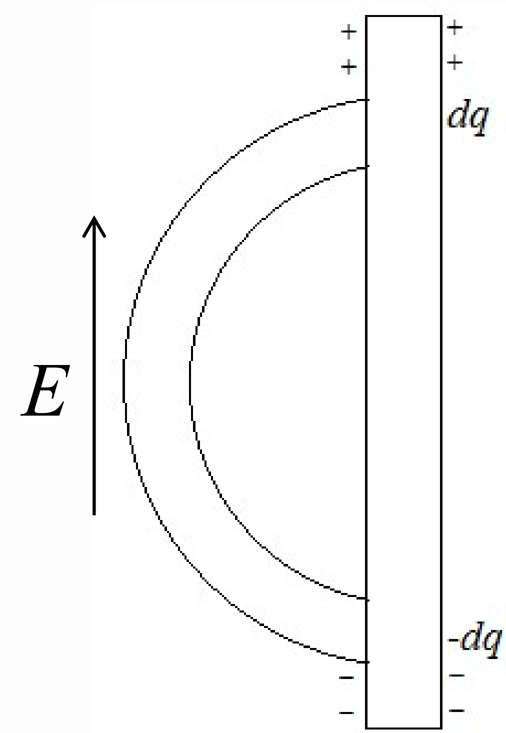
- Thinner laminate with higher ε_r preferred for feed line

- Smaller values of temperature coefficient of dielectric constant desired to ensure lower centre-frequency variations

- Thus choice of material is a balancing act!

Artificial dielectrics

- First proposed by Kock (1948)
- A 2- or 3- dimensional lattice of conducting elements reproduce processes occurring in the molecules of a natural dielectric
- Polarization in conductors:



A thin cylindrical rod in an E field becomes a dipole.

- Requirements for constructing artificial dielectrics:
  - Element size to be smaller than the minimum wavelength to avoid resonance effects
  - Spacing to be less than 1 λ to avoid diffraction effects
- Analysis similar to the one for natural dielectrics

- For a square array, the relative permittivity can be shown to be

No. of elements/unit area of plane normal to axes of elements

$$\varepsilon_r = 1 + \frac{\alpha_o N / \varepsilon_o}{1 - \alpha_o N / 2\varepsilon_o}$$

Free-space polarizability; depends on the shape and material of the elements

- The above equation is equivalent to Clausius-Mossotti relation

- Applications:
  - Antenna performance improvement
  - Microwave lenses
  - Radomes

Summary

- Dielectrics – physical and mathematical concepts
  - Polarization
  - Dielectric constant
  - Loss tangent
  - Delay in dielectrics
  - Types

- Artificial dielectrics